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Some Secondary Effects in a Simple Piping Structure Under Heating

An analysis is made of an ell-shaped piping configuration subjected to uniform heating in order to assess the effect of secondary influences (axial deformation, shear, beam-column action, and difference between arc and chord) in relieving reactions.

Bending and torsion are the principal sources of relief from thermal expansion in most piping configurations. However, secondary influences, viz., (*a*) axial deformation, (*b*) shearing deformation, (*c*) beam-column action, and (*d*) shortening owing to the difference between arc and chord, may become significant in configurations one dimension of which greatly exceeds the other two. In the following, an analysis, including these secondary influences, is made of the most fundamental configuration, an ell shape.

Analysis

Considering the uniform tip-loaded cantilever shown in Fig. 1, one may verify that the expression

$$y = \{ML^2u(1 - \cos\xi)\sec u + PL^3(1 + \eta u^2)[\sin\xi + (1 - \cos\xi)\tan u] - PL^3\xi\}/EIu^3 \quad (1)$$

(see nomenclature) satisfies the conditions

$$EIy'' = M + Q(\Delta - y) + P(L - x) \quad (2)$$

$$y(0) = 0;\ y(L) = \Delta;\ y'(0) = \gamma;\ y'(L) = \varphi \quad (3)$$

where primes denote differentiation with respect to x and where

$$\Delta = [M\alpha_1 + PL(\alpha_2 + \eta\alpha_3)](L^2/EI) \quad (4)$$

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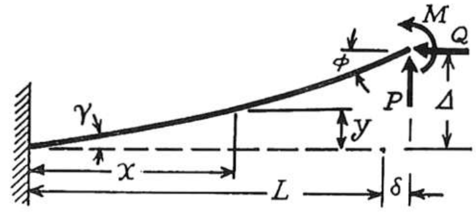


Fig. 1

$$\varphi = [M\alpha_3 + PL(\alpha_1 + \eta\alpha_4)](L/EI) \quad (5)$$

The coefficients α_i appearing in these equations and subsequently are shown in Table 1 where alternate forms are given to expedite computation.

Although the difference between arc and chord is neglected in determining lateral deflections (equation (2)), it is considered in determining axial deflections. Thus

$$\delta = eL - 2\nu pL\rho^2/E(1 - \rho^2) - (Q - A_F p)L/AE - \lambda \quad (6)$$

In equation (6) the first term on the right represents thermal expansion, the second represents axial contraction resulting from hoop tension caused by internal pressure, the third represents axial contraction due to axial compression in the metal, and the last represents the effective axial contraction resulting from the difference between arc and chord. The pressure terms may be combined to yield the equation

$$\delta = (e + pH)L - QL/AE - \lambda \quad (7)$$

The term λ is approximated in the usual way, noting, however, that that portion of slope resulting from shear deflection should not be included; discarding terms of third and higher order in small quantities, one obtains

Nomenclature

- a, b = length of straight pipe elements
- A = cross section area of pipe material
- A_F = cross section area of pipe contents
- e = unit thermal strain
- E = Young's modulus of elasticity
- F_1, F_2 = structural forces in ell-configuration
- g_{ij} = coefficients defined in equations (17)–(20)
- G = shearing modulus of elasticity
- H = $(1 - 2\nu)\rho^2/E(1 - \rho^2)$ = pressure coefficient
- I = moment of inertia of pipe-material cross section
- k_1 = $\pi^2/4$
- k_2 = $k_1 - \epsilon_2$
- L = length of pipe
- M, N = bending moments
- p = internal pressure
- P = lateral force
- Q = axial compressive force
- u = $w^{1/2}$
- v = $(-w)^{1/2}$
- w = QL^2/EI
- x = axial coordinate
- X = horizontal deflection of joint
- y = lateral coordinate
- Y = vertical deflection of joint
- α_i = coefficients (see Table 1)
- $\beta_{1,2}$ = coefficients defined in equation (15)
- γ = $\zeta P/AG$ = effective shearing strain
- δ = tip axial extension
- Δ = tip lateral deflection
- $\epsilon_{1,2}$ = conveniently small positive numbers
- ζ = shear-distribution factor (see equation (9))
- η = $\zeta EI/AGL^2$
- θ = rotation of joint
- λ = difference between arc and chord (first approximation)
- μ_1, μ_2 = coefficients in equations (9), (10), and (11)
- ν = Poisson's ratio
- ρ = (inside diameter of pipe)/(outside diameter of pipe)
- φ = tip rotation
- ξ = xu/L

Table 1 Formulas for coefficients in equations (4), (5), and (8)

	$\lvert w\rvert < \epsilon_1$	$w > \epsilon_1$	$w < -\epsilon_1$
α_1	$\frac{1}{2} + \frac{5}{24}w + \frac{61}{720}w^2 + \frac{277}{8064}w^3 + \ldots$	$(\alpha_4 - 1)/w$	
α_2	$\frac{1}{3} + \frac{2}{15}w + \frac{17}{315}w^2 + \frac{62}{2835}w^3 + \ldots$	$(\alpha_3 - 1)/w$	
α_3	$1 + w\alpha_2$	$\tan u/u$	$\tanh v/v$
α_4	$1 + w\alpha_1$	$\sec u$	$\mathrm{sech}\, v$
α_5	$\alpha_1 + \alpha_2 + w\alpha_1\alpha_2 - \frac{5}{12} - \frac{61}{360}w - \frac{277}{4032}w^2 - \frac{50521}{1814400}w^3 - \ldots$	$(\alpha_3\alpha_4 - 2\alpha_1)/w$	
α_6	$2\alpha_1 + w\alpha_1^2 - \frac{2}{5} - \frac{17}{105}w - \frac{62}{945}w^2 - \frac{17966}{675675}w^3 - \ldots.$	$(\alpha_3^2 - 3\alpha_2)/w$	
α_7	$(\alpha_4^2 + \alpha_3 - 2)$		
α_8	$(\alpha_3^2 - \alpha_2)$		
α_9	$(\alpha_5 + \eta\alpha_3\alpha_4)$		
α_{10}	$(\alpha_6 + 2\eta\alpha_8 + \alpha^2\alpha_7)$		

$$\lambda = \frac{1}{2}\int_0^L (y'^2 - \gamma^2)dx$$

$$= (M^2\alpha_8 + MPL\alpha_9 + P^2L^2\alpha_{10})(L^3/4E^2I^2) \quad (8)$$

Before proceeding further, one should note that the factor ζ which represents an effective shearing stress-concentration factor, may be evaluated[2] as

$$\zeta = \mu_1 + \mu_2\rho^2/(1 + \rho^2)^2 \quad (9)$$

$$\mu_1 = (7 + 14\nu + 8\nu^2)/6(1 + \nu)^2 \quad (10)$$

$$\mu_2 = (10 + 20\nu + 8\nu^2)/3(1 + \nu)^2 \quad (11)$$

for uniform wall circular pipe. If the wall is thin, then $\zeta \approx 2$.

Application to Ell-Shaped Configuration

The preceding analysis is applied to the ell-shaped configuration shown in Fig. 2, by separating into two parts (designated as part 1 and part 2) and identifying the quantities illustrated in Fig. 3 with those introduced earlier as indicated in Table 2. By equating expressions for X, Y, and θ, one obtains

$$ea - F_2a/AE - \lambda + paH = [-N\bar{\alpha}_1 + F_2b(\bar{\alpha}_2 + \bar{\eta}\bar{\alpha}_3)](b^2/EI) \quad (12)$$

$$eb - F_1b/AE - \bar{\lambda} + pbH = [-N\alpha_1 + F_1a(\alpha_2 + \eta\alpha_3)](a^2/EI) \quad (13)$$

$$Na\alpha_3 - F_1a^2(\alpha_1 + \eta\alpha_4) = -Nb\bar{\alpha}_3 + F_2b^2(\bar{\alpha}_1 + \bar{\eta}\bar{\alpha}_4) \quad (14)$$

where a superior bar indicates evaluations using the parameters of part 2 (see Table 2) and the absence of a bar indicates evaluation using the parameters of part 1. From equation (14), one obtains

$$N = \frac{F_1a^2(\alpha_1 + \eta\alpha_4) + F_2b^2(\bar{\alpha}_1 + \bar{\eta}\bar{\alpha}_4)}{a\alpha_3 + b\bar{\alpha}_3} = \beta_1F_1 + \beta_2F_2 \quad (15)$$

and thus has to deal with the system

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} (e + pH)a - \lambda \\ (e + pH)b - \bar{\lambda} \end{bmatrix} \quad (16)$$

where

$$g_{11} = -\bar{\alpha}_1\beta_1b^2/EI \quad (17)$$

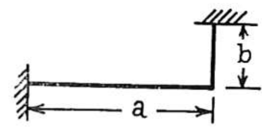
$$g_{12} = [b^3(\bar{\alpha}_2 + \bar{\eta}\bar{\alpha}_3) + aI/A - \bar{\alpha}_1\beta_2b^2]/EI \quad (18)$$

$$g_{21} = [a^3(\alpha_2 + \eta\alpha_3) + bI/A - \alpha_1\beta_1a^2]/EI \quad (19)$$

$$g_{22} = -\alpha_1\beta_2a^2/EI \quad (20)$$

[2] J. E. Brock, "Shear Distribution in Piping," *Heating, Piping, and Air Conditioning*, vol. 35, January, 1963, pp. 141–143.

Fig. 2



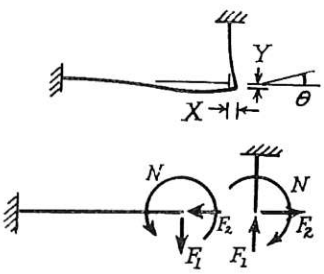


Fig. 3

Table 2 Identification of terms in ell-configuration

Item	Part 1	Part 2
L	a	b
P	$-F_1$	F_2
Q	F_2	F_1
M	N	$-N$
Δ	$-Y$	X
δ	X	Y
φ	θ	θ

Computation. A computational procedure is evident. One assumes initial values of F_1 and F_2 (these may be taken to be zero), evaluates the coefficients g_{ij} and the terms λ and $\bar{\lambda}$, and upon solving the system of equations (16), obtains improved values, continuing until satisfactory convergence is reached. A difficulty is encountered if partially converged values should exceed the Euler buckling load for either member. This is impossible physically and results in "jumping" to another branch of the trigonometric functions appearing in most of the coefficients α_i.[3] To assure convergence, one should "truncate" computed values of w in an appropriate fashion to assure that no computation is attempted with a value $w \geqq \pi^2/4$. To avoid computational difficulties, the author has used the following truncation scheme. Let ϵ_2 represent a convenient small number, let $k_1 = \pi^2/4$, and let $k_2 = k_1 - \epsilon_2$. Then if $w > k_2$, replace w by

$$w^* = (k_1w - k_2)/(w + k_1 - 2k_2) \quad (21)$$

[3] R. K. Livesley's formulation of a similar problem avoids this difficulty; see his paper, "Application of Electronic Digital Computer to Some Problems of Structural Analysis," in *The Structural Engineer*, vol. 34, 1956, pp. 1–12.

It is interesting to note that the effect of internal pressure is precisely analogous to increase of temperature [cf. equation (16)], except as the elastic and thermal constants are temperature dependent. In the latter event, the evaluation should use the Young's modulus corresponding to the temperature of the pipe, and the coefficient of expansion should be that for the pipe temperature and for zero stress. It is this "stressless" coefficient that is usually tabulated and it results from a little recognized thermodynamic relation that this stressless value should be used even though the material is under stress.[4]

[4] Postulating that strain e is a sufficiently differentiable function of stress σ and temperature T, only, one has

$$\frac{\partial}{\partial \sigma}\left[\frac{\partial e}{\partial T}\right] = \frac{\partial}{\partial T}\left[\frac{\partial e}{\partial \sigma}\right]$$

and interpreting the quantities appearing in brackets, one deduces that

$$\left[\frac{\partial \alpha}{\partial \sigma}\right]_T = \frac{d}{dT}\left(\frac{1}{E}\right)$$

where E is Young's modulus, well known to be a function of temperature, and where α is the coefficient of linear thermal expansion, seen thus to be a function of both stress and temperature. As a constrained structure is heated from a stressless initial condition, both E and α change with temperature and α changes with stress, as well. However, a process of partial integration shows that the final state may be arrived at by taking E as constant and equal to its value at the higher temperature and by evaluating thermal strain by integrating $\alpha(T, \sigma)$ with $\sigma = 0$.

A FORTRAN program, implementing the foregoing analysis, has been written for a large digital computer and appears to have been debugged successfully. It is planned to use this program to make a systematic investigation of the importance of the secondary effects under discussion in cases of technological interest, and the results of such a study will be reported elsewhere.